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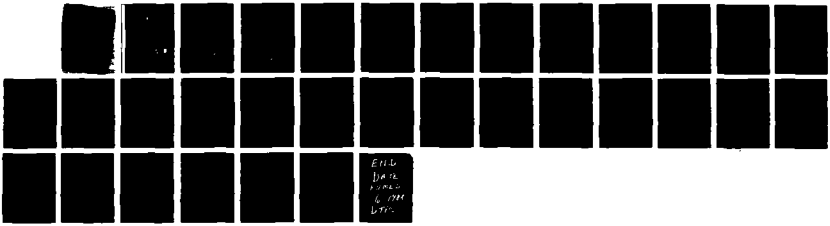

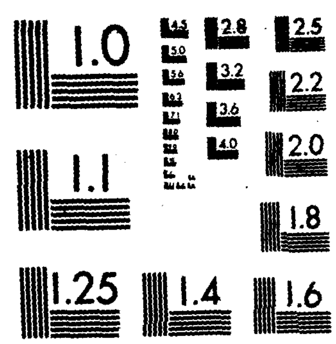
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MELBOURNE, VICTORIA

Aerodynamics Technical Memorandum 392

A MAXIMUM LIKELIHOOD PARAMETER ESTIMATION PROGRAM FOR GENERAL NON-LINEAR SYSTEMS (U)

by

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A MAXIMUM LIKELIHOOD PARAMETER ESTIMATION PROGRAM FOR GENERAL NON-LINEAR SYSTEMS (U)

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SUMMARY

A computer program has been developed for the Maximum Likelihood estimation of parameters in general non-linear systems. Sensitivity matrix elements are calculated numerically, overcoming the need for explicit sensitivity equations. Parameters such as break points and time shifts are successfully determined using both simulated and actual test data.

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CONTENTS

NOTATION

GENERAL :

dt	Time interval between successive time measurements, $t_{i+1} - t_i$
n	Measurement noise vector
N	Number of discrete time points
t	Time
$\hat{t}$	Time prior to time shifting
t_i	i^{th} discrete time point
u	Control input vector
x	State vector
y	Observation vector
y_j	j^{th} observation variable
z	Measurement vector
$\delta\xi_k$	Increment in ξ_k used for numerical sensitivity calculations
$\Delta\xi$	Change in ξ per iteration
ξ	Parameter vector
ξ_k	k^{th} parameter
$\nabla_\xi y$	Sensitivity matrix
$\nabla_\xi y_{jk}$	jk^{th} element of sensitivity matrix $(= \partial y_j/\partial \xi_k)$

LANDING GEAR DROP TESTS (SECTION 3)

C_1	Tyre "spring" constant
d	Oleo deflection
d_{max}	Maximum value of d
d^*	Tyre compression
d_o	Break point
d_1, d_2	First, second stage oleo deflection (two stage type)
g	Acceleration due to gravity
G_1, G_2	Oleo damping terms
K_1, K_2	Oleo "spring" constants
L	Load on landing gear
L_o, L_t	Load on oleo, tyre
M	Landing gear mass
w	$\dot{d} + \dot{d}^*$
w_o	Initial drop velocity
τ_d, τ_L	Time shifts in d, L

FLIGHT TEST (SECTION 4)

a_n	Normal acceleration
c	*Reference chord*
C_M	Moment coefficient, as a function of C_{M_α}, C_{M_q}, C_{M_δ}, C_{M_o}, and $C_{M_{\dot\alpha}}$
C_N	Force coefficient, as a function of C_{N_α}, C_{N_q}, C_{N_d}, and C_{N_o}

g	Acceleration due to gravity
I_y	Moment of inertia about pitch axis
m	Aircraft mass
q	Rate of change of angle of pitch
$\bar{q}$	Dynamic pressure
R	Radian to degree conversion factor
S	Wing area
V	Aircraft velocity
X_{α}, X_{a_n}	Longitudinal instrument offsets from c. g.
Z_{a_n}	Vertical instrument offset from c. g.
α	Angle of attack
δ	Elevator deflection
θ	Angle of pitch

1. INTRODUCTION

The Maximum Likelihood Parameter Estimation technique is widely used to determine aircraft flight parameters from flight test data (Ref. 1). Much effort in this field has been confined to analysis of linear systems. If the model is non-linear, the problem becomes more difficult.

Here at Aeronautical Research Laboratory (ARL), a Maximum Likelihood computer program was developed (Ref. 2) to solve such non-linear problems. This program was successfully used to determine accelerometer offsets and calibration errors, given dynamic flight test data (Ref. 3). However, a sizable proportion of the program was problem-specific, in particular the evaluation of the sensitivity matrix. Sensitivity matrix elements were calculated explicitly by mathematical differentiation of the state equations, often a long and tedious process.

In this document, a Maximum Likelihood Parameter Estimation computer program for general non-linear systems is described. Sensitivities are calculated numerically by finite differences, overcoming the need for explicit sensitivity equations. This method allows the user to select, as unknown parameters, quantities such as break points and time shifts, for which sensitivities are not known in explicit form.

Other Maximum Likelihood programs representing generalized non-linear systems are available (Ref. 4). However, their specific application to the estimation of break points or time shifts does not appear to have been reported. The program described here is successfully used to determine both of these quantities.

Section 2 provides a brief theoretical description of the Maximum Likelihood method for non-linear systems, and the procedure used here to obtain sensitivities numerically. In Section 3, the computer program developed is validated using simulated data, before being applied to a study of aircraft landing gear modelling, a topic currently being investigated by the Aircraft Behaviour Studies - Rotary Wing Group at ARL. In Section 4, the program is used to estimate time shifts in measured flight test data.

2. DESCRIPTION OF METHOD

Assume that the system can be described in general by a set of non-linear dynamic equations of the form :

$$\dot{x}(t) = f(\, x(t),\, u(t),\, \xi \,) \qquad [1]$$

$$y(t) = g(\, x(t),\, u(t),\, \xi \,) \qquad [2]$$

$$z(t_i) = y(t_i) + n(t_i) \qquad [3]$$

where :

x is the state vector

u is the control input vector

y is the observation vector

z is the measurement vector, sampled at N discrete time points, t_i, for i = 1,.. N

n is the measurement noise vector, assumed to be Gaussian with zero mean

ξ is the vector of unknown parameters

The Maximum Likelihood method (described in Ref. 2) determines the most probable value of ξ by an iterative procedure which can be summarized as follows :

$$R = \frac{1}{N}\sum_{i=1}^{N}[z(t_i) - y(t_i)]\,[z(t_i) - y(t_i)]^T \qquad [4]$$

$$\Delta\xi = \left[\sum_{i=1}^{N}(\nabla_\xi y(t_i))^T R^{-1} (\nabla_\xi y(t_i))\right]^{-1} \left[\sum_{i=1}^{N}(\nabla_\xi y(t_i))^T R^{-1} (z(t_i) - y(t_i))\right] \qquad [5]$$

where R is the covariance of residuals and $\Delta\xi$ is the change in ξ per iteration.

Given R and ξ, we obtain $\Delta\xi$ and hence an improved value of ξ, which is used to obtain a new $y(t_i)$ and thus improved R. This process is repeated until convergence is achieved. Computation of $\Delta\xi$ requires at each time point, t_i :

i) values of the measurement vector, $z(t_i)$

ii) values of the observation vector, $y(t_i)$

iii) the sensitivity matrix, $\nabla_\xi y(t_i)$

Values of the measurement vector are read in as data. Computation of the current observation vector, $y(t_i)$, from [2] requires current state vector values, which are obtained by numerical integration of the assumed system state equations ([1] above). A fourth-order Runge-Kutta numerical integration procedure is adopted. The sensitivity matrix elements are here approximated by numerical differences. This approach overcomes the need for explicit sensitivity equations, which are not always easy to determine. The central difference method is adopted here, requiring evaluation of state and observation variables at two perturbed parameter values, $\xi+\delta\xi$ and $\xi-\delta\xi$. The jkth element of the sensitivity matrix, $\nabla_\xi y(t_i)_{jk}$, is given by :

$$\nabla_\xi y(t_i)_{jk} = \frac{\partial y_j}{\partial x_k} = \frac{y_j(\xi_k+\delta\xi_k) - y_j(\xi_k-\delta\xi_k)}{2\delta\xi_k} \qquad [6]$$

where y_j and ξ_k are components of vectors y and ξ.

The program developed here is based on an earlier ARL Maximum Likelihood program (Ref. 2). Details of program changes are listed in Appendix A.

3. EXAMPLES USING SIMULATED DATA

Simulated data is used to examine a number of different systems. In 3.1, the numerical sensitivity matrix computation is validated by analysing a simple system, both with explicit and numerical sensitivity matrix computations. In 3.2 and 3.3, the numerical model is used to examine more complicated systems which include the effect of break points and time shifts.

3.1 Validation of Numerical Sensitivity Matrix Computation

Consider the case of an aircraft landing gear drop test. The landing gear, comprising a large mass, M, attached to an oleo and tyre, is dropped with initial velocity w_0. The oleo is modelled as a massless, non-linear damped spring, and the tyre as a massless, linear undamped spring.

The load on the oleo, L_o, is given by :

$$L_o = K_1 d^2 + G_1 \dot{d} \qquad [7]$$

where d is the oleo deflection. The load on the tyre, L_t, is given by :

$$L_t = C_1 d^* \qquad [8]$$

where d^* is the tyre compression. Since the tyre and oleo are assumed to be massless, the oleo load is equal to the tyre load, thus $L_o = L_t \equiv L$ (see Fig. 1).

The equation of motion for the system is :

$$M(\ddot{d} + \ddot{d}^*) = Mg - L \qquad [9]$$

Taking d, d^*, and w ($= \dot{d} + \dot{d}^*$) as state variables, the following state equations are obtained :

$$\dot{w} = g - \frac{C_1 d^*}{M} \qquad [10]$$

$$\dot{d} = \frac{C_1 d^* - K_1 d^2}{G_1} \qquad [11]$$

$$\dot{d}^* = w - \dot{d} \qquad [12]$$

Observation variables are the load and oleo deflection, given by :

$$L_{obs} = C_1 d^* \qquad [13]$$

$$d_{obs} = d \qquad [14]$$

There are no control inputs, u, in this example. The unknown parameters are K_1, C_1, and G_1.

The above equations, [10] - [14], are subject to constraints that d, d^*, and L are all ≥ 0.

The landing gear is dropped with initial velocity w_o, so initial values of w, d, and d^* are w_o, 0, and 0 respectively.

Using simulated time histories of oleo load and deflection, with 81 data points and $\Delta t = 0.01$ s, the Maximum Likelihood method is applied with a) explicit sensitivity matrix equations and b) numerical sensitivity matrix computations. Zero mean noise with an RMS of 0.0025 m for d and 0.5 kN for L is superimposed on the simulated data.

3.1.1 Explicit Sensitivity Matrix Calculation

Sensitivity matrix elements, $\partial y_i/\partial \xi_k$, are given by d_{K1}, d_{C1}, d_{G1}, L_{K1} ($= C_1 d_{K1}^*$), L_{C1} ($=C_1 d_{C1}^* + d^*$) and L_{G1} ($= C_1 d_{G1}^*$) where $d_{K1} = \partial d/\partial K_1$ etc. Sensitivities are obtained from partial derivatives of the state equations [10] - [12] :

$$\dot{d}_{K1} = \frac{1}{G_1}\left(C_1\, d_{K1}^* - 2dK_1 d_{K1} - d^2 \right) \qquad [15]$$

$$\dot{d}_{K1}^* = w_{K1} - \dot{d}_{K1} \qquad [16]$$

$$\dot{w}_{K1} = \frac{-C_1 d_{K1}^*}{M} \qquad [17]$$

$$\dot{d}_{C1} = \frac{1}{G_1}\left(d^* + C_1 d_{C1}^* - 2dK_1 d_{C1} \right) \qquad [18]$$

$$\dot{d}_{C1}^* = w_{C1} - \dot{d}_{C1} \qquad [19]$$

$$\dot{w}_{C1} = \frac{-1}{M}\left(C_1 d_{C1}^* + d^2 \right) \qquad [20]$$

$$\dot{d}_{G1} = \frac{1}{G_1}\left(C_1 d_{G1}^* - 2dK_1 d_{G1} - \frac{C_1 d^*}{G_1} + \frac{K_1 d^2}{G_1} \right) \qquad [21]$$

$$\dot{d}_{G1}^* = w_{G1} - \dot{d}_{G1} \qquad [22]$$

$$\dot{w}_{G1} = \frac{-C_1 d_{G1}^*}{M} \qquad [23]$$

The above derivatives are numerically integrated with respect to time using a fourth-order Runge Kutta procedure, and the sensitivity elements calculated, from which the most probable values for parameters K_1, C_1, and G_1 are determined.

After 10 iterations, excellent agreement with the simulated data is obtained (Fig. 2). The parameter values are listed in Table 1 with the *true values* being the values used in the simulation, and the *a priori values* being the initial guess of these values. The Cramer - Rao error bounds are also shown.

Table 1. Parameter Estimates for 3-Parameter Model

Parameter	Unit	A Priori Value	True Value	Maximum Likelihood Parameter Values	
				Explicit Sensitivities	Numerical Sensitivities
K_1	Nm^{-2} ($\times 10^5$)	1	4	3.993±0.009	3.993±0.009
G_1	Nsm^{-1} ($\times 10^4$)	1	2.5	2.496±0.007	2.496±0.007
C_1	Nm^{-1} ($\times 10^5$)	1	7	7.037±0.041	7.038±0.041
Run Time (s)				4.29	11.62

3.1.2 Numerical Sensitivity Matrix Computation

Sensitivity matrix elements are calculated using the central difference method (see [6]) with $\delta\xi_k/\xi_k = 10^{-3}$. After 10 iterations, the Maximum Likelihood method results in the parameter values shown in Table 1. It is seen that results are almost identical to the explicit sensitivity case, and indeed graphs of oleo deflection and load versus time (Fig. 3) are found to be indistinguishable from the explicit sensitivity graphs (Fig. 2). There is however an increase in computer run time due to the numerical integration of the state equations for incremented parameters $\xi + \delta\xi$ and $\xi - \delta\xi$, as well as the usual ξ.

Other test cases (not shown here) were also found to give excellent agreement between explicit and numerical sensitivity calculations, thus validating the numerical procedure used here for the computation of the sensitivity elements.

The effect of varying the size of $\delta\xi_k/\xi_k$ in the numerical sensitivity case was examined. For $\delta\xi_k/\xi_k$ ranging from 0.1 down to 10^{-4}, no appreciable increase in accuracy was obtained for the simulated data of Fig. 2 (see Table 2).

Table 2. Maximum Likelihood Parameter Estimates using Numerical Sensitivity Calculations with Varying $\delta\xi_k/\xi_k$

Parameter	Unit	A Priori Value	True Value	$\delta\xi_k/\xi_k$ 0.1	0.01	0.001	0.0001
K_1	Nm^{-2} ($x10^5$)	1	4	3.9928	3.9928	3.9928	3.9928
G_1	Nsm^{-1} ($x10^4$)	1	2.5	2.4962	2.4962	2.4962	2.4962
C_1	Nm^{-1} ($x10^5$)	1	7	7.0380	7.0376	7.0376	7.0381

3.2 Systems with Break Points

A more advanced landing gear system is the two-stage type. For oleo deflection, d, greater than some value, d_o, a second, stiffer, more damped "spring" is activated. In this way, extra hard landings are catered for, whilst soft landings do not suffer from overdamping. Essentially, a different set of state equations exist for $d > d_o$ with d_o termed as a *break point.*

Assume the oleo load to be modelled as :

$$L = C_1 d^* = \begin{cases} K_1 d^2 + G_1 \dot{d} & (\text{for } d < d_o) \\ K_1 d_o^2 + K_2 (d-d_o)^2 + G_2 \dot{d} & (\text{for } d \geq d_o) \end{cases}$$

The state equations in this case are :

$$\dot{w} = g - \frac{C_1 d^*}{M} \qquad [25]$$

$$\dot{d} = \frac{C_1 d^* - K_1 d_1^2 - K_2 d_2^2}{G} \qquad [26]$$

$$\dot{d}^* = w - \dot{d} \qquad [27]$$

where :

$$\left.\begin{aligned} d_1 &= d \\ d_2 &= 0 \\ G &= G_1 \end{aligned}\right\} d < d_o \qquad \left.\begin{aligned} d_1 &= d_o \\ d_2 &= d - d_o \\ G &= G_2 \end{aligned}\right\} d \geq d_o$$

d_1 is the first stage deflection and d_2 the second stage deflection. Observation variables are the oleo load and deflection.

Along with parameters K_1, K_2, G_1, G_2 and C_1, the break point d_o is taken as an unknown parameter. Using simulated time histories of oleo load and deflection, the Maximum Likelihood method with numerical sensitivity calculations is applied. The calculated observations give excellent agreement with the simulated measurements (Fig. 3). The parameter values after 10 iterations are given in Table 3.

Table 3. Parameter Estimates for 6-Parameter Model (Including Break Point)

Parameter	Unit	A Priori Value	True Value	Maximum Likelihood Parameter Value
K_1	Nm^{-2} ($\times 10^5$)	2	4	4.040 ± 0.017
K_2	Nm^{-2} ($\times 10^5$)	10	45	45.825 ± 1.348
G_1	Nsm^{-1} ($\times 10^4$)	1.5	2.5	2.473 ± 0.009
G_2	Nsm^{-1} ($\times 10^4$)	3.5	4	3.966 ± 0.030
C_1	Nm^{-1} ($\times 10^5$)	4	7	7.064 ± 0.039
d_o	m	0.1	0.23	0.227 ± 0.001

It should be noted that careful consideration needs to be given to the selection of the a priori value of the break point parameter d_o. Clearly, if $d_o > d_{max}$ (where d_{max} is the maximum value reached by quantity d in the time interval under consideration), either initially or during one of the early iterations (when parameter values are liable to oscillate), then any small change in d_o will have no effect on the observation vector, **y**. Consequently, sensitivities $\partial y_1/\partial d_o$ will all be zero resulting in no improved estimate of d_o. A suitable initial value for d_o can usually be obtained by experimentation.

For most parameters, the size of $\delta\xi_k$ used in the numerical sensitivity calculations, should be significantly less than the size of ξ_k. We use here $\delta\xi_k/\xi_k = 10^{-3}$. However, break points or time shifts are special cases, and careful consideration needs to be given to the size of $\delta\xi_k$. This is because measurements, **z**, only exist at discrete time points, t_i, and observations, **y**, are only calculated at these time points. For break point d_o (time shifts are discussed in Section 3.3), in order that $\partial y_i/\partial d_o$ is non-zero for at least one t_i, any change, δd_o, in the value of d_o must be large enough such that the time where the break point acts, $t(d_o)$, moves across at least one time point. If $t(d_o)$ is within the time interval $[t_i, t_{i+1}]$, then for δd_o to result in non-zero sensitivities, we require:

$$|t(d_o) - t(d_o - \delta d_o)| \quad > \quad t(d_o) - t_i$$

or $$|t(d_o + \delta d_o) - t(d_o)| \quad > \quad t_{i+1} - t(d_o)$$ (see Fig. 5)

The size of δd_o necessary to satisfy the above condition can be found by experimentation. In the case reported here, $\delta d_o = 0.01$ was sufficient.

3.3 Systems with Time Shifts

When recording actual test data, time shifts between different measurements can occur as a result of instrumentation lag (Ref. 3). In general, the time shift for a particular measurement is not known, and can be included in the Maximum Likelihood procedure as an additional parameter. Time shifts are expected to be small and a priori values are usually set to zero.

In a landing gear drop test such as that in Section 3.2, any time shifts in measurements *d* and *L* need not be the same and can be represented by unknown parameters τ_d and τ_L respectively.

The simulated drop test data from Section 3.2 is used, with time shifts added in. Assuming the same landing gear model as in Section 3.2, we aim to determine these time shifts using the Maximum Likelihood procedure.

The state and observation equations (Section 3.2) are :

$$\dot{d}(\hat{t}) = \frac{C_1 d^*(\hat{t}) - K_1 d_1^2(\hat{t}) - K_2 d_2^2(\hat{t})}{G} \qquad [28]$$

$$\dot{d}^*(\hat{t}) = w(\hat{t}) - \dot{d}(\hat{t}) \qquad [29]$$

$$\dot{w}(\hat{t}) = g - \frac{C_1 d^*(\hat{t})}{M} \qquad [30]$$

$$d_{obs}(\hat{t}) = d(\hat{t}) \qquad [31]$$

$$L_{obs}(\hat{t}) = C_1 d^*(\hat{t}) \qquad [32]$$

where $\hat{t}$ is the time *without* any time shift.

A time shift in *d* or *L* will result in a translation of the above observations :

$$d_{obs}(t) = d(\hat{t} - \tau_d) \qquad [33]$$

$$L_{obs}(t) = L(\hat{t} - \tau_L) \qquad [34]$$

Application of the Maximum Likelihood procedure gives excellent results in 10 iterations (Fig. 6 and Table 4) with both time shifts accurately predicted. Run time has increased from 24.13 to 30.50 seconds; however, this is to be expected since the number of unknown parameters has increased from 6 to 8.

Table 4. Parameter Estimates for 8-Parameter Model (Including Break Point and Time Shifts)

Parameter	Unit	A Priori Value	True Value	Maximum Likelihood Parameter Value
K_1	Nm^{-2} ($\times10^5$)	2	4	4.031 ± 0.028
K_2	Nm^{-2} ($\times10^5$)	20	45	47.535 ± 1.977
G_1	Nsm^{-1} ($\times10^4$)	1.5	2.5	2.486 ± 0.018
G_2	Nsm^{-1} ($\times10^4$)	3	4	3.931 ± 0.054
C_1	Nm^{-1} ($\times10^5$)	5	7	7.039 ± 0.110
d_o	m	0.1	0.23	0.229 ± 0.002
τ_d	s	0	0.07	0.070 ± 0.0008
τ_L	s	0	0.09	0.090 ± 0.0005

In the Maximum Likelihood program developed here, any parameter representing a time shift is automatically rounded to a multiple of dt, the time interval between successive measurements, t_i. Observations are only computed at time t_i for comparison with the measurements, and thus any time shift must of necessity be a multiple of dt. Likewise, the small increment in time lag parameter $\delta\tau$ *required for the numerical* evaluation of sensitivities, must also be a multiple of dt. Sensitivities are evaluated here by examining two outputs that have time lags $\tau + dt$ and $\tau - dt$. Then :

$$\frac{\partial y_i}{\partial \tau} = \frac{y_i(\tau+dt) - y_i(\tau-dt)}{2dt}$$

4. RESULTS USING FLIGHT TEST DATA

Fixed wing flight test data was available for an aircraft undergoing a longitudinal manoeuvre. State variables are taken as α, q, and θ with control input δ. Observation variables, for which measurements are given, are α, q, and a_n where :

α = angle of attack (deg)

θ = pitch angle (deg)

q = rate of change of pitch angle, $\dot{\theta}$ (deg s^{-1})

a_n = normal acceleration (ft s^{-2})

δ = elevator deflection (deg)

The full state equations for a longitudinal manoeuvre are given in Equation [55], Ref. 5. For small α and ϕ, we obtain the state equations :

$$\dot{\alpha} = \frac{-\bar{q}SR}{mV}\left(C_N + \dot{\alpha}_o\right) + q + \frac{gR}{V}\cos\theta \quad [35]$$

$$\dot{q} = \frac{\bar{q}ScR}{I_y}C_M \quad [36]$$

$$\dot{\theta} = q \quad [37]$$

with :

$$C_N = C_{N_\alpha}\alpha + C_{N_q}\frac{qc}{2VR} + C_{N_\delta}\delta + C_{N_o} \quad \textit{(Force Coefficient)}$$

$$C_M = C_{M_\alpha}\alpha + C_{M_q}\frac{qc}{2VR} + C_{M_\delta}\delta + C_{M_o} + C_{M_{\dot{\alpha}}}\frac{\dot{\alpha}c}{2VR} \quad \textit{(Moment Coefficient)}$$

Observations (Equation [56], Ref. 5) are :

$$\alpha_{obs} = \alpha - \frac{X_\alpha}{V}q \quad [38]$$

$$q_{obs} = q \quad [39]$$

$$a_{n_{obs}} = \frac{\bar{q}S}{mg}C_N + \frac{X_{a_n}}{gR}\dot{q} + \frac{Z_{a_n}}{R^2g}q^2 \quad [40]$$

where :

R = Radian to degree conversion factor = 57.2958 deg rad^{-1}

S = Wing area = 550 ft^2

m = Aircraft mass = 1929.88 slug

V = Aircraft velocity, assumed constant, = 1088.4199 ft s^{-1}

g = Acceleration due to gravity = 32.174 ft s^{-2}

c = Reference chord = 8.8 ft

I_y = Moment of inertia about pitch axis = 310 912.906 slug ft^2

X_α = α - vane Longitudinal offset from c. g. = 26.3065 ft

Z_{a_n} = Accelerometer (normal) vertical offset from c. g. = 1.3685 ft

X_{a_n} = Accelerometer (normal) longitudinal offset from c. g. = 3.3715 ft

C_{N_q} = 8.75 rad^{-1}

C_{N_δ} = 0.015 deg^{-1}

$\bar{q}$ = Dynamic pressure, assumed constant, = 1318.026 lbf ft^{-2}

with initial conditions $\alpha_o = 2.3169$ deg, $q_o = 0.0948$ deg s^{-1}, and $q_o = 2.4568$ deg. Unknown parameters are C_{M_α}, C_{M_q}, C_{M_δ}, C_{M_o}, C_{N_α}, C_{N_o}, and $C_{N_o} + \dot{\alpha}_o$. Parameter $C_{M_{\dot{\alpha}}}$ is also unknown, but in the manoeuvre reported here, it is difficult to determine both $C_{M_{\dot{\alpha}}}$ and C_{M_q} independently. Consequently, the two parameters are linked by a factor, obtained from their a priori values :

$$C_{M_{\dot{\alpha}}} = 0.2836 \times C_{M_q}$$

The Maximum Likelihood procedure is applied *without* time shifts, with a time interval of dt = 1/60 sec and 10 iterations, and with the control input variation shown in Fig. 7. Results are shown in Fig. 8 and Table 5. It is noted that there are discrepancies between actual measurements and predicted observations which appear to be due to time lags. Application of the Maximum Likelihood procedure with three additional time shift parameters, τ_α, τ_q, and τ_{a_n}, leads to marked improvements in the predicted observation variables (Fig. 8). The Cramer-Rao error bounds are also reduced (Table 5) indicating improved parameter identification.

Table 5. Parameter Estimates using Flight Test Data

Parameter	Unit	A Priori Value	Maximum Likelihood Parameter Values	
			Without Time Shifts	With Time Shifts
C_{M_α}	deg^{-1}	-0.050	-0.036 ± 0.0002	-0.041 ± 0.00008
C_{M_q}	rad^{-1}	-30	-9.92 ± 0.46	-8.29 ± 0.14
C_{M_δ}	deg^{-1}	-0.035	-0.019 ± 0.0002	-0.022 ± 0.00009
C_{M_o}	-	0	0.102 ± 0.0007	0.116 ± 0.0003
C_{N_α}	deg^{-1}	0.09	0.071 ± 0.0009	0.071 ± 0.0003
C_{N_o}	-	0	-0.093 ± 0.003	-0.092 ± 0.0008
$C_{N_o} + \dot{\alpha}_o$	-	-0.25	-0.087 ± 0.004	-0.093 ± 0.001
τ_α	s	0	-	0.033 ± 0.001
τ_q	s	0	-	0.050 ± 0.0007
τ_{a_n}	s	0	-	0.067 ± 0.0007

5. CONCLUDING REMARKS

A general Maximum Likelihood program, suitable for identification of parameters in non-linear systems, has been described. A summary of the theoretical background has been given, and differences between an earlier ARL program noted. Sensitivities are now calculated numerically, a process which allows time shifts or break points to be identified. The program was validated on a simple 3-parameter system, before being applied to a study of aircraft landing gear modelling. The study, using simulated data, has shown that the program

can successfully identify time shifts, break points, and conventional parameters, using the numerical sensitivity approach. Finally, the program has been applied to actual flight test data, and has successfully matched time shifts and conventional parameters.

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APPENDIX A

Computer Program Structure

The program described in this document was developed from an earlier ARL program, *COMPAT* (Ref. 2). In its earlier form, *COMPAT* required four problem-dependent subroutines, which calculated :

- Initial conditions
- Explicit equations for derivatives of sensitivities
- Derivatives of state equations
- Output responses (observations)

In the modified program, *COMPAT.JB.5,* initial conditions are now specified in the data file, *COMDAT.JB,* and sensitivity matrix elements are evaluated numerically, requiring only two problem-dependent subroutines (*RESP.JB* and *DERIV.JB.3* which calculate the output responses and state derivative equations respectively). The final program structure is :

Main program :	*COMPAT.JB.5*	
Subroutines :	*RESP.JB*	(output response equations)
	DERIV.JB.3	(derivatives of state vector equations)
	COM.SUB.JB	(Maximum Likelihood iterative procedure)
Data file :	*COMDAT.JB*	(input the necessary information to specify the problem, eg. number of states, number of parameters , as well as a priori parameter values and initial state conditions. Finally, time histories of control inputs, $u(t_i)$, and measurements, $z(t_i)$, are read in)
Output files :	*HP1.PLOT*	(contains time histories of the inputs, and measured & calculated outputs suitable for producing time history plots)
	COMOUT.JB.3	(contains an iterative history, as well as final values of parameters and their Cramer-Rao error bounds)

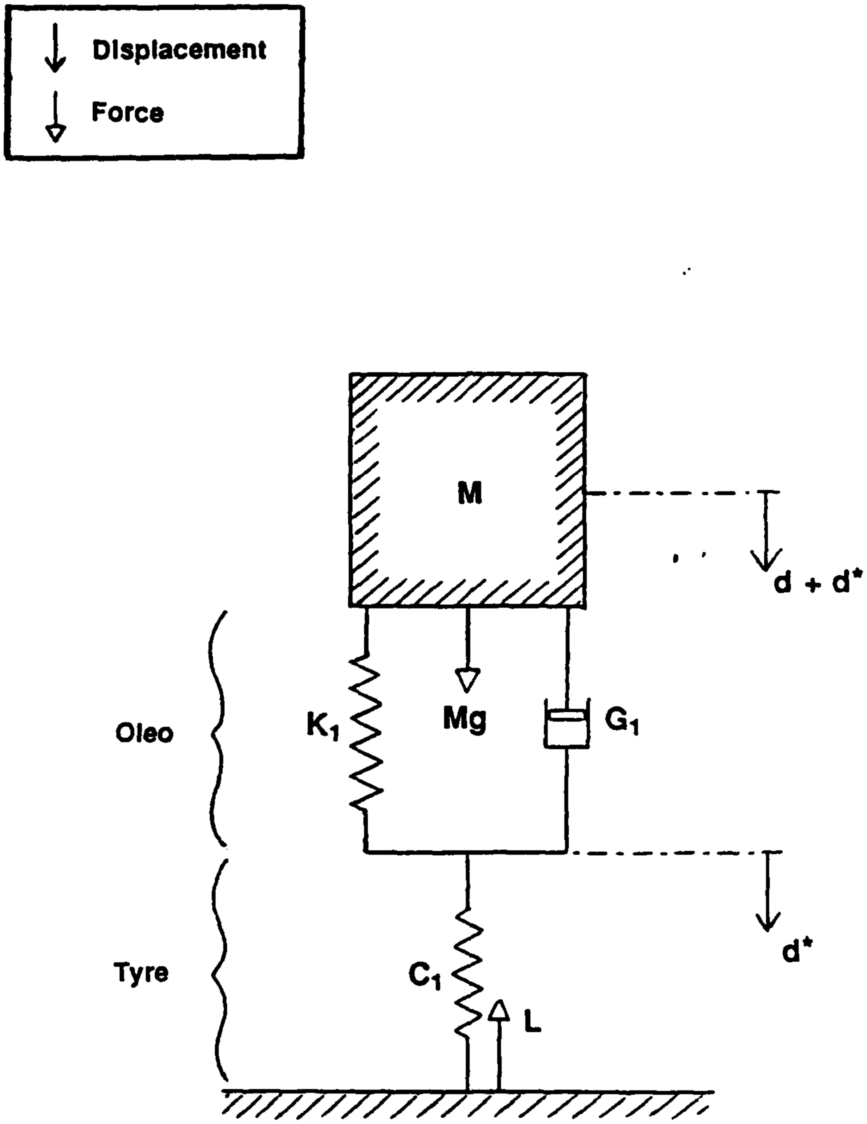

Figure 1. Forces and Displacements on Landing Gear

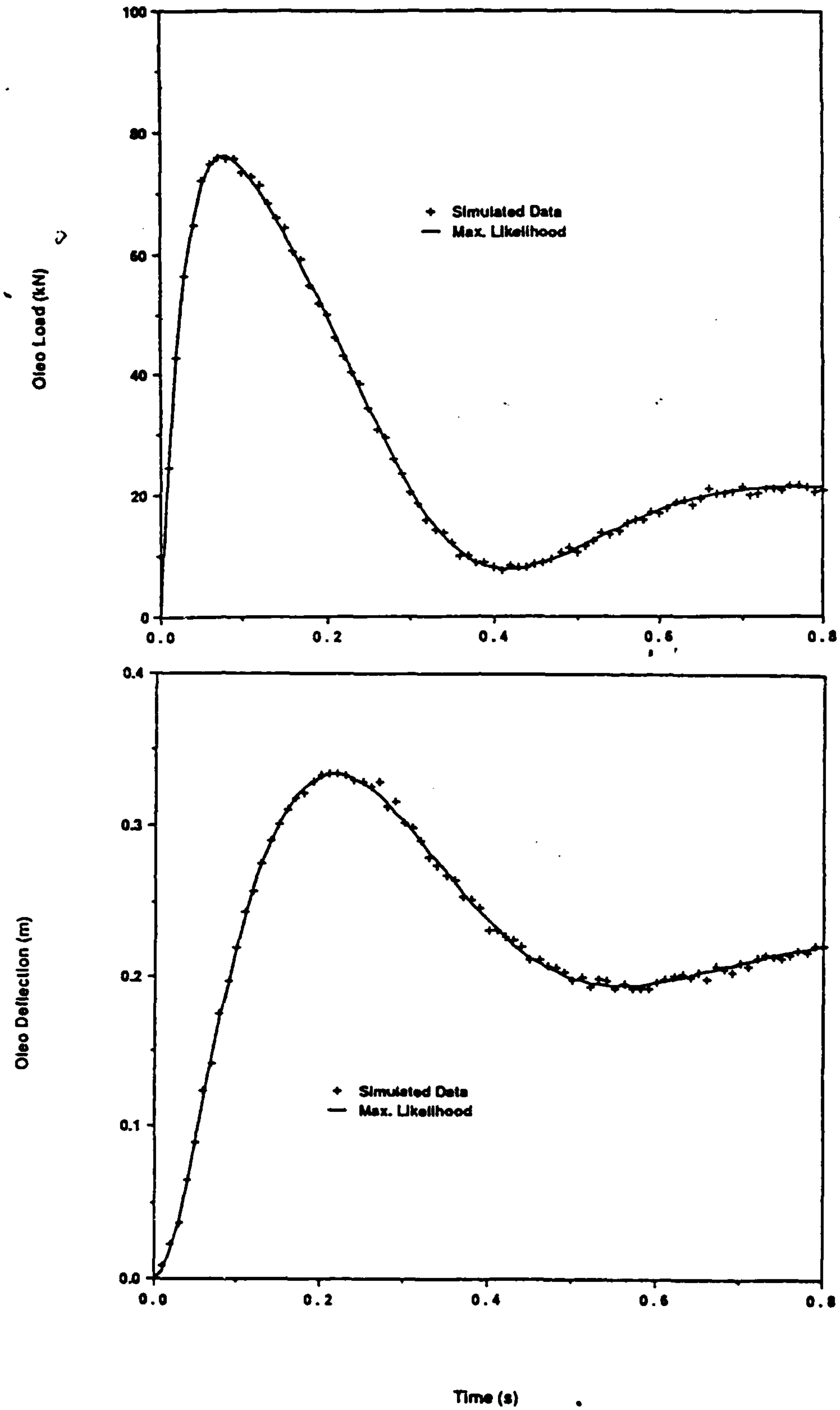

Figure 2. Oleo Load and Displacement for 3-Parameter Model (Explicit Sensitivity Equations) $w_o = 4\ ms^{-1}$, M=2000 kg

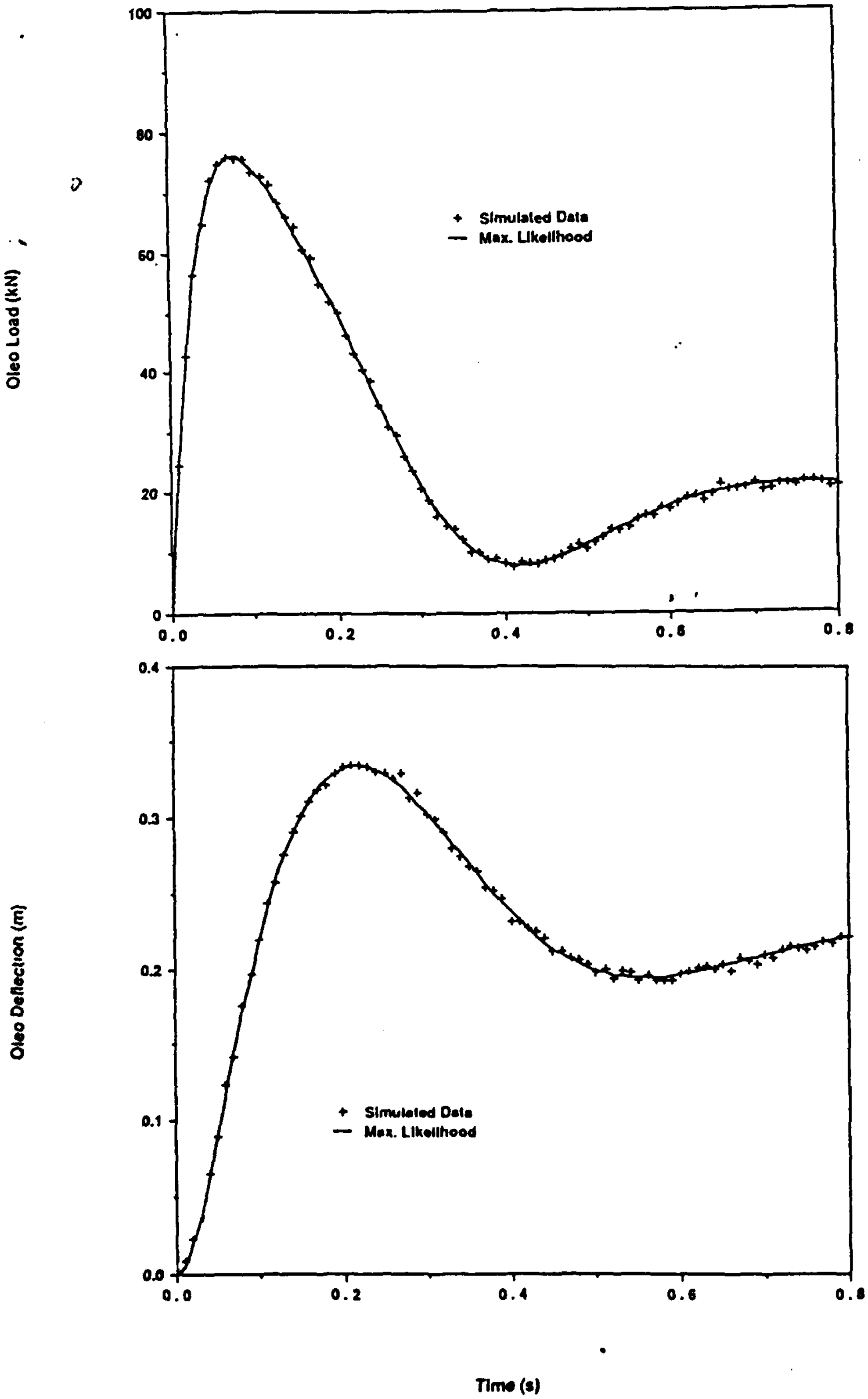

Figure 3. Oleo Load and Displacement for 3-Parameter Model (Numerical Sensitivity Computations) $w_o = 4\ ms^{-1}$, M=2000 kg

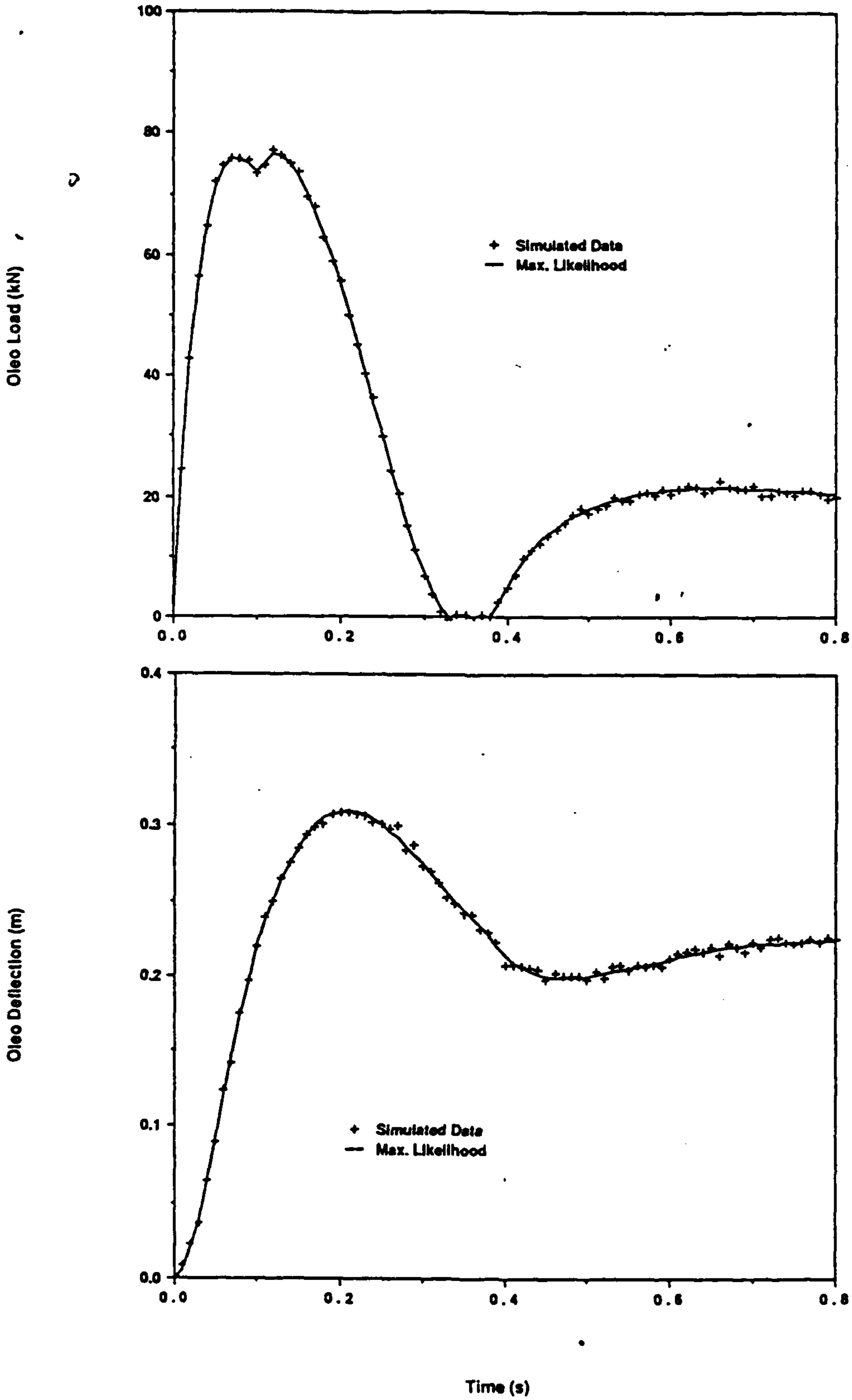

Figure 4. Oleo Load and Displacement for 6-Parameter Model (Including Break Point)
$w_o = 4\ ms^{-1}$, M=2000 kg

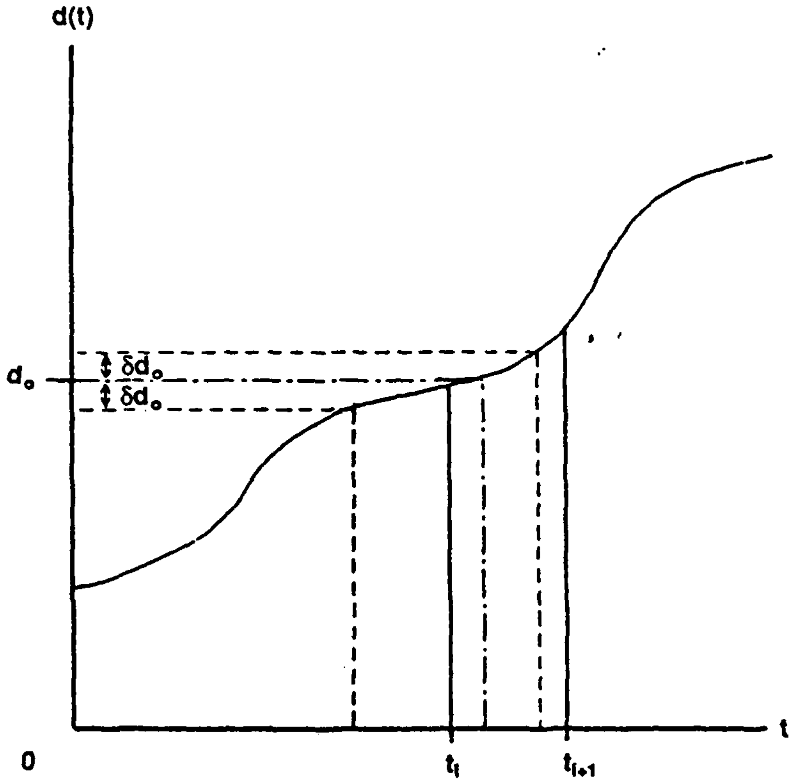

Figure 5. The Effect of Shifting a Break Point, d_o, by δd_o

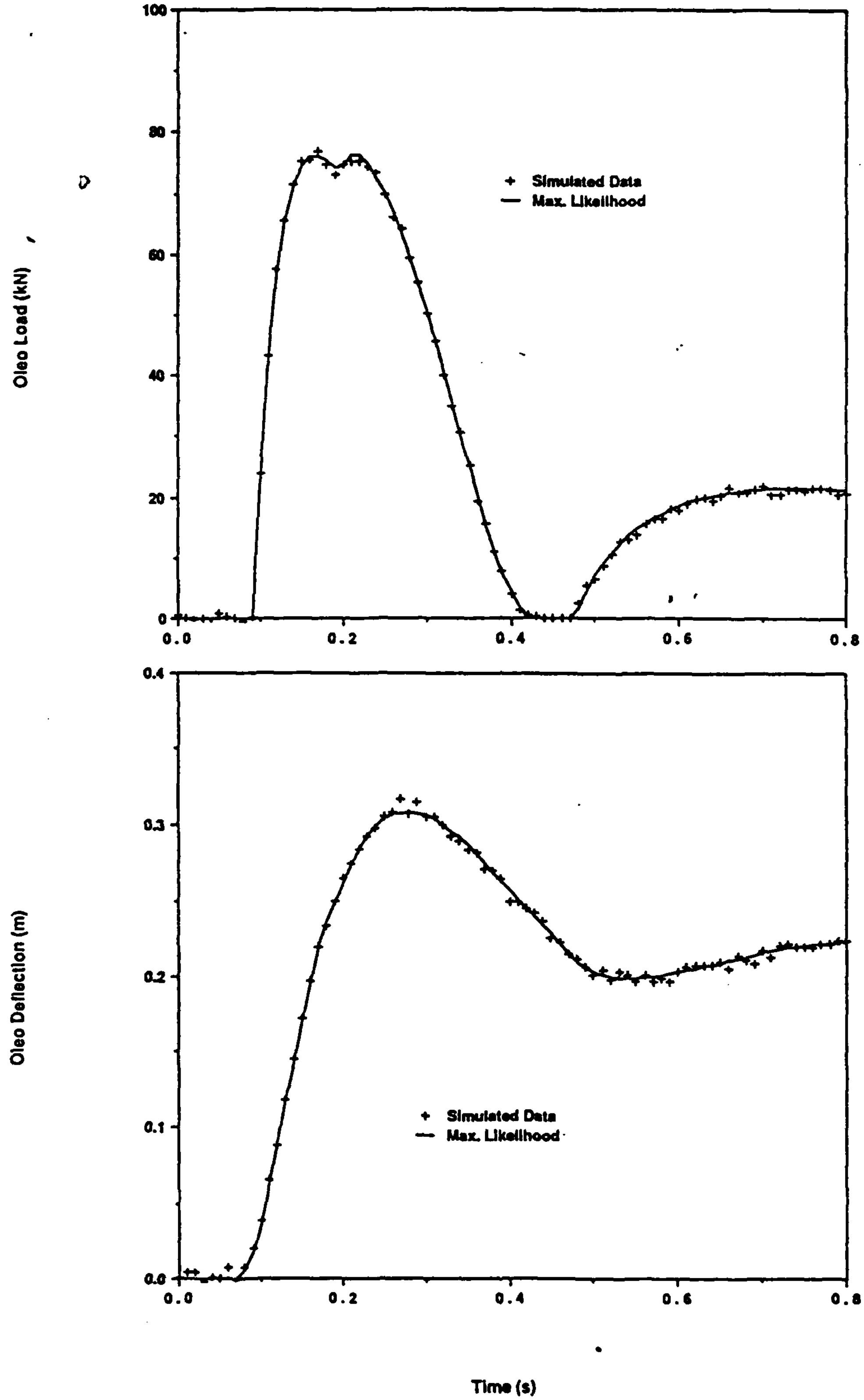

Figure 6. Oleo Load and Displacement for 8-Parameter Model (Including Break Point and Time Shifts) $w_o = 4\ ms^{-1}$, M=2000 kg

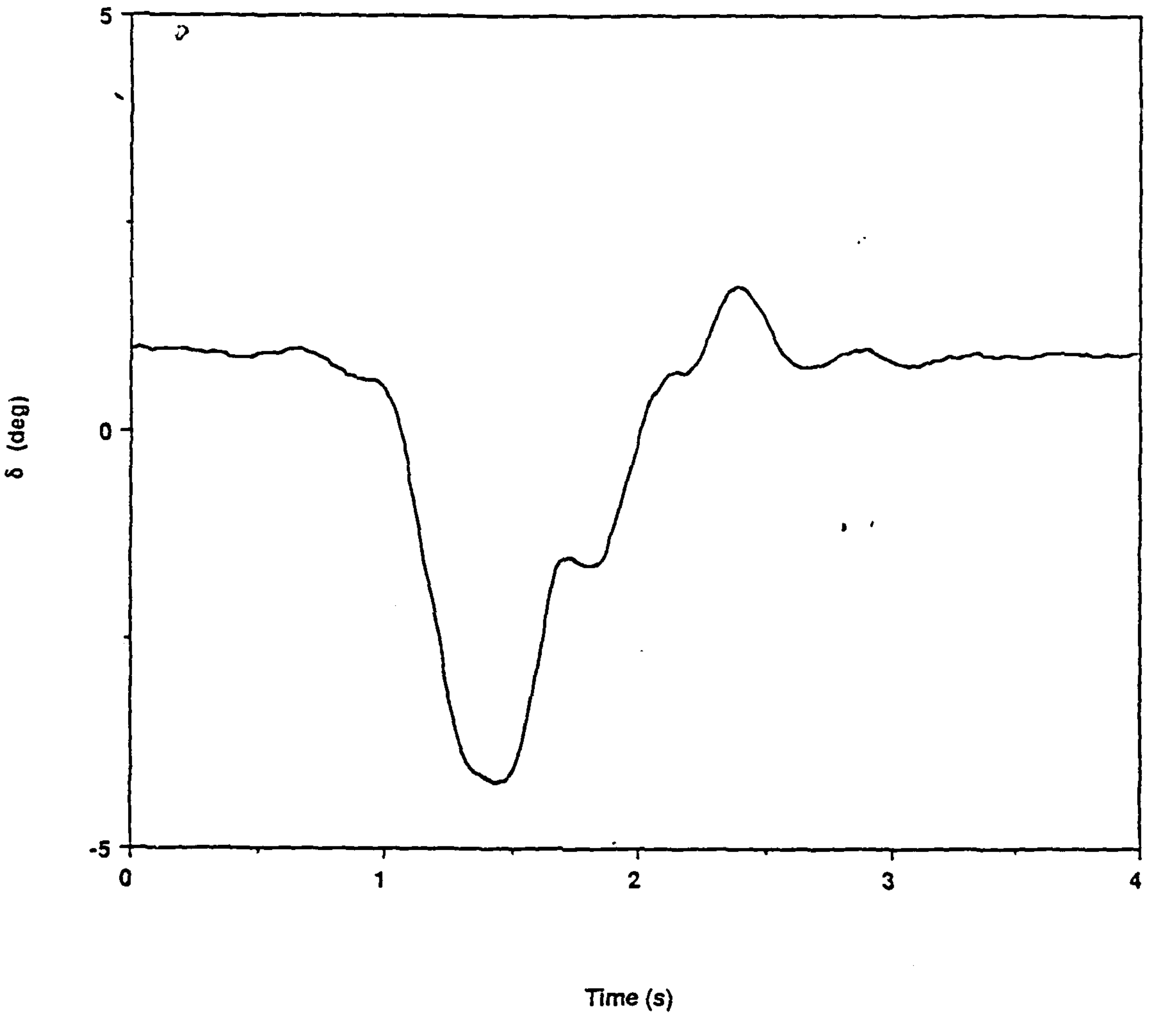

Figure 7. Control Input (Elevator Deflection) Variation with Time - Flight Test

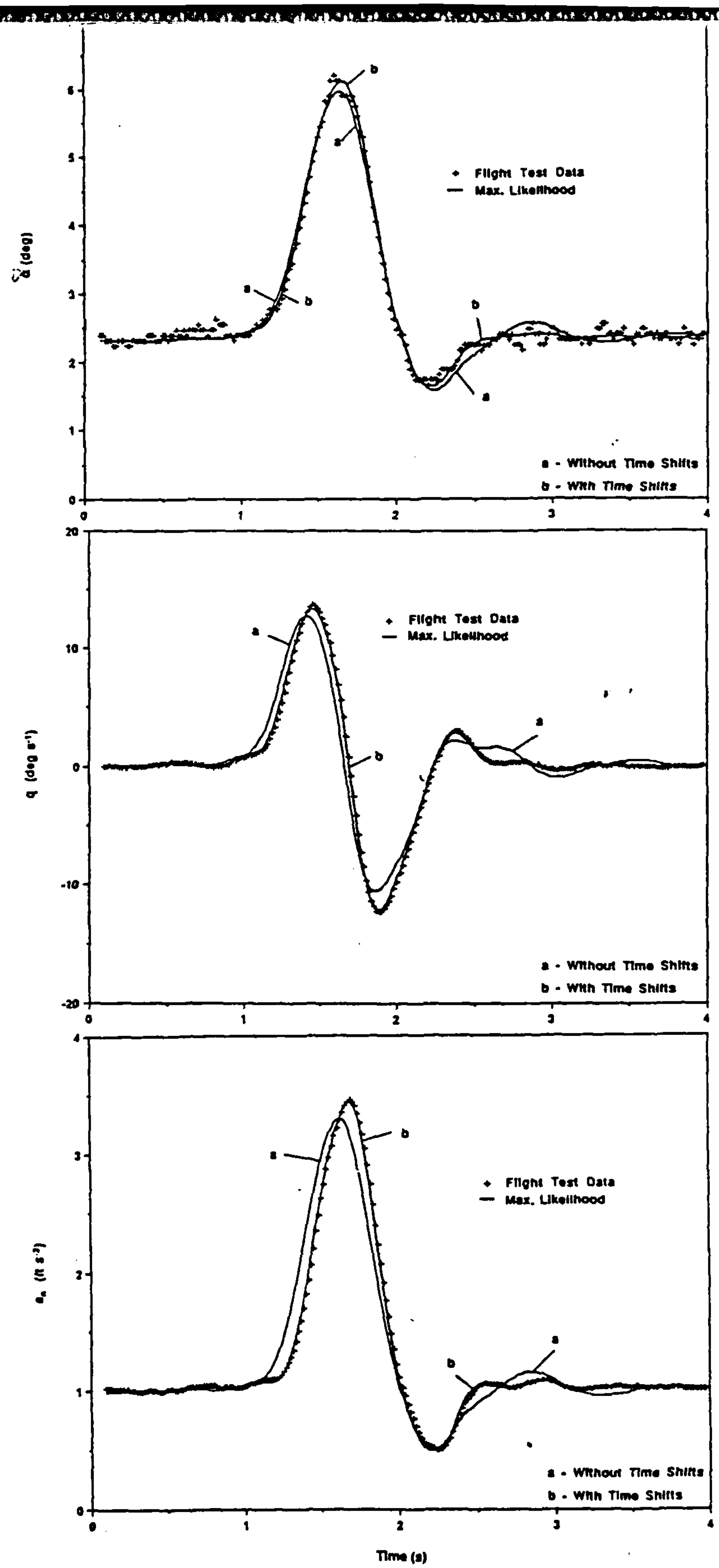

Figure 8. Variation of Angle of Attack, Pitch Rate, and Normal Acceleration - Flight Test

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